Centripetal Acceleration

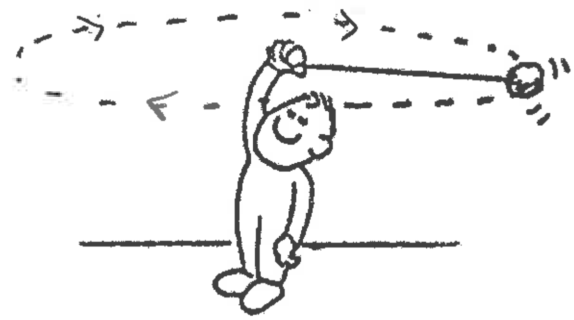
We will now investigate the special case of objects which exhibit uniform circular motion – motion in a circular path at a constant speed.

First let's consider a mass on a string being twirled in a horizontal circle at a constant speed. Let's determine the speed of the object. Remember that speed is defined as:

$$v = \frac{d}{t}$$

We define the period of motion (T) as the time it takes to complete one rotation. How far does it travel in one rotation?

circumference



We can find the circumference of the circular path (distance traveled) by:

$$c = 2\pi r$$

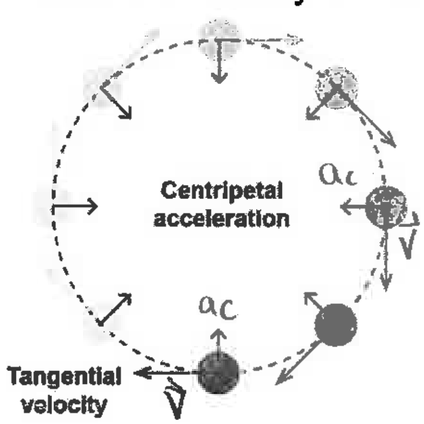
Therefore the speed of an object in uniform circular motion is:

$$v = \frac{2\pi r}{T} \leftarrow \text{period}$$

Ok so we've figured out its speed, but is the mass accelerating? Remember that the mass is traveling at a constant speed. However, acceleration is defined as:

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

So how does the velocity of the mass change with respect to time?



Notice that the direction of the velocity at any time is …
tangent to the circle

And the acceleration of the object is always …
towards the center

This is the definition of centripetal - which means
center seeking!

So even though it may be traveling at a constant speed anything traveling in a circular path is accelerating because the direction of its velocity is always changing.

The acceleration of an object in uniform circular motion can be found using:

$a_c = \frac{v^2}{r}$ Where: a_c = centripetal acceleration v = speed of the object r = radius of the path	$a_c = \frac{4\pi^2 r}{T^2}$ Where: T = period (s)	$a_c = 4\pi^2 r f^2$ Where: f = frequency (Hz)

Example: A car traveling at 14 m/s goes around an unbanked (flat) curve that has a radius of 96 m. What is its centripetal acceleration?

Given: $v = 14\,m/s$
$r = 96\,m$

$$a_c = \frac{v^2}{r} = \frac{(14\,m/s)^2}{96\,m} = \boxed{2.04\,m/s^2}$$

Example: A plane makes a complete circle with a radius of 3282 m in 2.0 min. What is the centripetal acceleration of the plane?

Given: $r = 3282\,m$
$T = 2.0\,min \times \frac{60\,s}{min} = 120\,s$

$$a_c = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2(3282\,m)}{(120\,s)^2} = \boxed{9.0\,m/s^2}$$

Example: The centripetal acceleration at the end of an electric fan blade has a magnitude of 1.75 x 10^3 m/s^2. The distance between the tip of the blade and the center of the fan is 12 cm. Calculate the frequency and period of rotation for the fan.

Given: $a_c = 1.75 \times 10^3\,m/s^2$
$r = 12\,cm = 0.12\,m$

$$a_c = 4\pi^2 r f^2$$

$$f^2 = \frac{a_c}{4\pi^2 r}$$

$$f = \sqrt{\frac{a_c}{4\pi^2 r}}$$

$$= \sqrt{\frac{(1.75\times10^3\,m/s^2)}{4\pi^2(0.12\,m)}}$$

$$= \pm 19.2\,Hz$$ (only +ve root though)

$$\boxed{= +19\,Hz}$$

$$T = \frac{1}{f}$$

$$= \frac{1}{19.2\,Hz}$$

$$\boxed{= 5.2\times10^{-2}\,s}$$