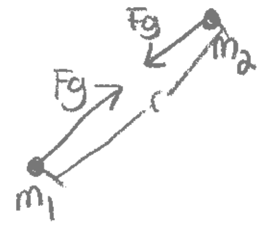
# Universal Gravitation

Newton, using the movement of planetary bodies as his guide, formulated the Universal Law of Gravitation which states there is a gravitational attraction between any two objects depending on their masses and the distance between them.

$$\boxed{F_g = \frac{G m_1 m_2}{r^2}}$$




Where $G$ = gravitational constant = $6.67 \times 10^{-11}$ N·m$^2$/kg$^2$
m = masses, measured in kg
r = distance between centre of masses, measured in m

Example: Determine the force of gravitational attraction between the earth (m = $5.98 \times 10^{24}$ kg) and a 70.0 kg physics student if the student is standing at sea level, a distance of $6.38 \times 10^6$ m from earth's center.

$$F_g = \frac{G m_1 m_2}{r^2} = \frac{6.67\times10^{-11}\,\frac{N\cdot\cancel{m^2}}{\cancel{kg^2}}\,(5.98\times10^{24}\,\cancel{kg})(70\,\cancel{kg})}{(6.38\times10^{6}\,\cancel{m})^2}$$

$$= \boxed{686\,N}$$

Note:
1. Gravity is universal. ALL objects attract each other with a force of gravitational attraction!

2. Mass matters! $F_g \propto m_1 m_2$ as m↑, so does Fg → directly proportional
If m doubles, Fg doubles

3. The inverse-square law $F_g \propto \frac{1}{r^2}$ as r↑, Fg↓ → inversely proportional
If r doubles, Fg↓ by 4 ($2^2$)

The Value of g

At Earth's surface at sea level we know $g = 9.8$ m/s$^2$ so redoing the example...

$$F_g = mg = 70.0\,kg \cdot 9.8\,m/s^2 = 686\,N$$

## Gravitational Fields

Scientists had difficulty explaining how two objects that are not in contact can exert a force on one another. In order to help conceptualize how this can occur, we had invented the idea of FIELDS.

A field is defined as...

an area of influence

To help imagine how these fields work, consider a campfire.

It seems as though the fire is emitting a heat field.

As you approach the fire the ...

field strength increases

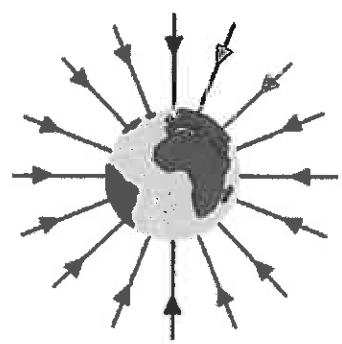
As you increase the size of the fire the ...

field strength increases

Just like this so-called heat field, gravitational fields surround any mass. Fields can be described as either vector or Scalar.

While heat is measured by temperature (a scalar) its field is also scalar.

Gravitational fields are force fields and as such are vector.



Vector fields, like vector quantities, are represented by arrows. In this case, the density of the arrows represents the magnitude of the field strength...

We are already quite familiar with gravitational field strength by its other name: acceleration due to gravity

Recall that: Fg = mg

Therefore $g = \frac{F_g}{m}$

Where g = gravitational field strength

This formula works fine if we stay put on Earth, but it falls way short once we leave Earth's surface because... g varies with distance!

However, we can derive a more useful formula:

$F_g = m_2 g$ und $F_g = \frac{G \overset{m_{earth}}{m_1} m_2}{r^2}$

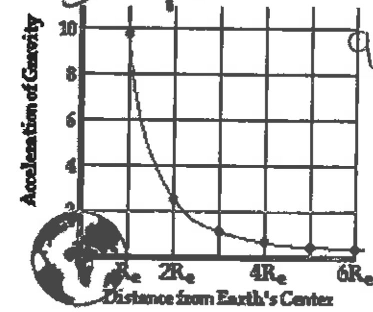
$\cancel{m_2} g = \frac{G m_{Earth} \cancel{m_2}}{r^2}$

$$\boxed{g = \frac{G m_{Earth}}{r^2}}$$

Thus, the value of g depends on location – inverse square law!

same as $g \propto \frac{1}{r^2}$ $\quad 9.8 = \frac{1}{r^2} = \frac{1}{2^2}$

$9.8 = \frac{1}{4}$




if an object moves to 2x earth's radii then

$$g = \frac{G\,m_{Earth}}{r^2} = \frac{6.67\times10^{-11}\,Nm^2kg^{-2} \cdot 5.98\times10^{24}\,kg}{(1.276\times10^{7}\,m)^2}$$

$$= 2.45\ N/kg \text{ or } 2.45\ m/s^2$$

Example: Calculate the value of g on the International Space Station which orbits at an altitude of roughly 400 km above Earth's surface.

$$g = \frac{6.67\times10^{-11}\,Nm^2kg^{-2}\ 5.98\times10^{24}\,kg}{(6.38\times10^{6}\,m + 4\times10^{5}\,m)^2} = 8.68\,m/s^2$$

$400\,km \times \frac{1000\,m}{1\,km} = 400000\,m$

Surprised at the answer? If ~90 percent of Earth's gravity reaches the space station, then why do astronauts float there?

The answer is because they are in __free fall__. Gravity is causing all objects to fall at the same rate, resulting in appearance of __weightlessness__, or what is known as __microgravity__.

Calculating g on other planets

The value of g on any other planet can be calculated from the mass of the planet and the radius of the planet. The equation takes the following form:

$$g = \frac{G \cdot m_{planet}}{R_{planet}^2}$$

Example: What is the gravitational field strength on the surface of the Moon, given a mass of $7.35\times10^{22}$ kg and a radius of $1.74\times10^{6}$ m?

$$g = \frac{G\,m_{planet}}{R_{planet}^2} = \frac{(6.67\times10^{-11})(7.35\times10^{22})}{(1.74\times10^{6})^2} = 1.62\,m/s^2$$

Example: Determine the value of g on Mercury, given it has a radius of $2.43 \times 10^{6}$ m and a mass of $3.2 \times 10^{23}$ kg.

$3.61\,m/s^2$