Centripetal Force

Whenever an object is accelerated there must be a...

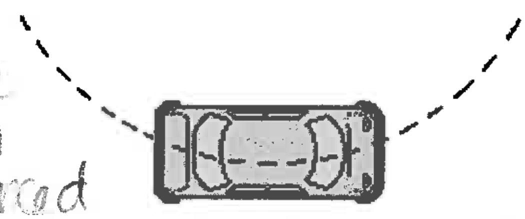
net force

This force is known as centripetal force, F_c, when acceleration occurs in a circular direction. This is not a new force, it is simply the net force that accelerates an object towards the center of its circular path.

To understand the centripetal force, it is important to Newton's first law of motion - the law of inertia. The law of inertia states that...

an object in motion stays in motion with the same speed and direction unless acted upon by an unbalanced force

Car in Motion Makes a Left-Hand Turn



A passenger in motion would remain in motion with the same speed and in the same direction, thus causing the "sensation of an outwards acceleration."

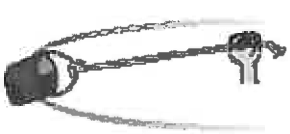
...meaning, moving objects will tend to travel in straight lines

Unless... acted upon by an unbalanced force

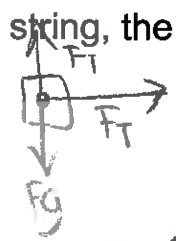
Thus, an unbalanced force, the centripetal force, is required for objects to move in circles.

Again, it is important to realize that the centripetal force is NOT a distinctly separate force. Centripetal is simply an adjective. This net force can be supplied by any force or combination of forces, but it must be directed towards the center of the circle for any object moving along a curved or circular path.

Examples:

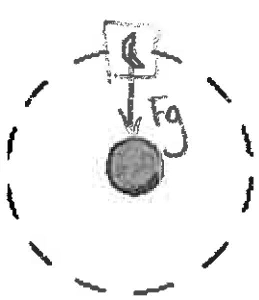


1) A mass is twirled in a circle at the end of a string, the centripetal force is provided by tension, T



2) When a car rounds a corner on a highway, the centripetal force is provided by friction, F_f

$\Sigma F_x = F_c = ma$



3) When the Moon orbits the Earth, the centripetal force is provided by gravity, F_g

Newton's Second Law we can help us to determine a formula for centripetal force:

$F_c = m \cdot a_c$ or $F_c = \frac{mv^2}{r}$ or $F_c = m\frac{4\pi^2 r}{T^2}$ or $F_c = m4\pi^2 r f^2$

Where:
F_c = centripetal force, N
m = mass, kg
a_c = centripetal acceleration, m/s²

Example: A string can withstand a force of 135 N before breaking. A 2.0 kg mass is tied to the string and whirled in a horizontal circle with a radius of 1.10 m. What is the maximum speed that the mass can be whirled at before the string breaks?

Given: $T = 135N = F_c$
$m = 2 kg$
$r = 1.10 m$

$F_c = \frac{mv^2}{r}$

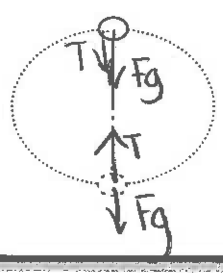
$v^2 = \frac{F_c r}{m}$

$v = \sqrt{\frac{F_c r}{m}}$

$v = \sqrt{\frac{135 N \cdot 1.10 m}{2.0 kg}}$

$= 8.62 m/s$

$= 8.6 m/s$



How does this differ for a mass moving in a vertical circle? Let's draw the forces at the top and bottom of the path.

As with any object moving in a circle there is a net force acting on it.

At the top of its arc the centripetal force (or net force) is: $F_c = T + F_g$

At the bottom of the arc the centripetal force is: $F_c = T - F_g$

Example: A 1.7 kg object is swung from the end of a 0.60 m string in a vertical circle. If the time of one revolution is 1.1 s, what is the tension in the string:

a) at the top?

Given $m = 1.7 kg$
$T = 1.1 s$
$r = 0.60 m$

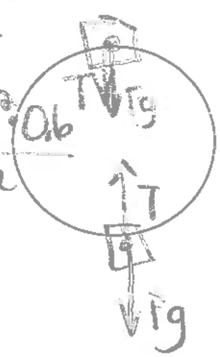
need: T $F_c = T + F_g$

$T = F_c - F_g$
$= 33.3 - mg$
$= 33.3 - 1.7(9.8)$
$= 16.6 N$
$= 17 N$

$F_c = m\frac{4\pi^2 r}{T^2}$

$= \frac{1.7 kg \cdot 4\pi^2 (0.6)}{1.1^2}$

$= 33.3 N$



b) at the bottom?

$T = F_c + F_g$
$= 33.3 + 16.6$
$= 5.0 \times 10^1 N$

Now suppose the mass is spun with just enough speed to keep it moving in a circular path (meaning constant speed). What is the tension in the string at the top?

$T = 0$

We say that the mass at the peak of the arc is weightless, because the net force working on it is only ___gravity___. This is the same as an object in total free fall. (meaning object as a projectile)

Thus, for the special case of finding the minimum speed of an object undergoing constant speed at the top of its circular arc we can use the equation:

$F_c = F_g$

$\frac{mv^2}{r} = mg$

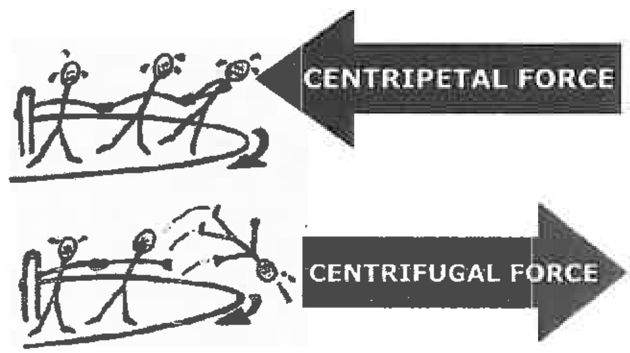
$v = \sqrt{gr}$

Example: An object is swung with constant speed in a vertical circle with a radius of 0.75 m. What is the minimum speed of the object at the top of the motion for the object to remain in circular motion?

$v = \sqrt{(9.8)(0.75)}$

$= 2.7\ m/s$

One last note: don't get centrifugal (the f-word!), which means center- ___fleeing___ confused with centripetal!



The centrifugal force does not actually exist. It is simply the apparent force that causes a rotating object to move in a straight line. In reality what we seem to feel as centrifugal force is really…

inertia!

Example: