Satellite Motion

A satellite is an object or body that revolves around another body that usually has much ore mass than the satellite.

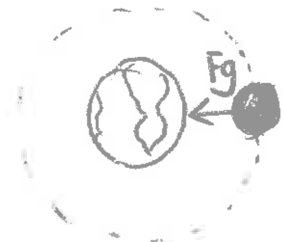
Two types: 1. Natural satellites like planets, moons

2. Artificial satellites such as GPS satellites
RADARSAT 1-2
International Space Station

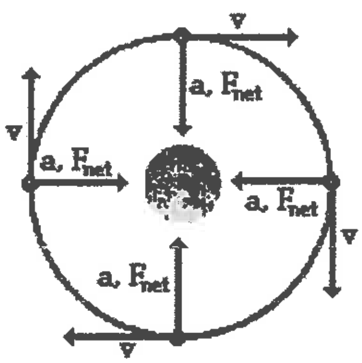
Satellites are <u>projectiles</u>, meaning the only force acting upon them is gravity.

A satellite of the Earth, such as the moon, is constantly falling. But it does not fall towards the Earth, rather it falls <u>around</u> the Earth.

Consider the Moon:

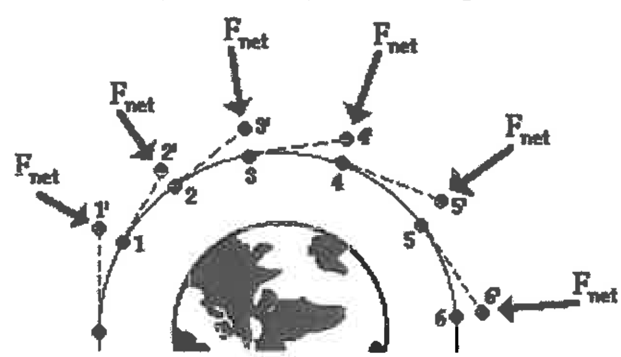


The motion of an orbiting satellite can be described by the same motion characteristics as any object in circular motion.

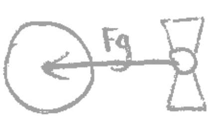


Satellites encounter inward forces and accelerations and tangential velocities.

An Orbiting Satellite Requires a Centripetal Force



<u>Example</u>: A 4500 kg Earth satellite has an orbital radius of 8.50×10^7 m. At what speed does it travel?



if $F_c = F_g$

$$\frac{\cancel{m_2} v^2}{\cancel{r}} = \frac{G m_1 \cancel{m_2}}{r^{\cancel{2}}}$$

$$v^2 = \frac{G m_1}{r}$$

$$\boxed{v = \sqrt{\frac{G m_1}{r}}}$$

$$v = \sqrt{\frac{(6.67\times10^{-11})(5.98\times10^{24})}{8.50\times10^7 m}}$$

$= 2200 m/s$

Example: The International Space Station orbits the Earth at an altitude of about 350 km above Earth's surface. (a) Determine the speed needed by the ISS to maintain its orbit.

$$v = \sqrt{\frac{Gm_1}{r}} = \sqrt{\frac{Gm_1}{r_E + h_{ISS}}}$$

$$= \sqrt{\frac{(6.67\times10^{-11}\ Nm^2kg^{-2})(5.98\times10^{24}\ kg)}{(6.38\times10^6\ m + 3.5\times10^5\ m)}}$$

$$= 7.698\times10^3\ m/s \quad \text{or} \quad 7.7\times10^3\ m/s$$

(b) Determine the orbital period of the ISS in minutes.

$$T = \frac{2\pi r}{v} = \frac{2\pi(6.73\times10^6\ m)}{7.698\times10^3\ m/s} \times \left(\frac{1\ min}{60\ s}\right) = 92\ min$$

$v = \frac{d}{t}$ so $t = \frac{d}{v}$ in this case $T = \frac{2\pi r}{v}$

Example: A satellite orbits the Earth at a radius of 2.20x10^7 m. What is its orbital period?

$$F_c = F_g$$

$$F_g \quad \cancel{m}a_c = \cancel{m}g$$

$$a_c = g$$

$$\frac{4\pi^2 r}{T^2} = \frac{Gm_1}{r^2}$$

$$\frac{T^2 Gm_1}{r^2} = 4\pi^2 r$$

$$T^2 Gm_1 = 4\pi^2 r^3$$

$$T = \sqrt{\frac{4\pi^2 r^3}{Gm_1}}$$

$$= \sqrt{\frac{4\pi^2(2.20\times10^7\ m)^3}{(6.67\times10^{-11})(5.98\times10^{24}\ kg)}}$$

$$= 32500\ s$$

The orbital speed of a satellite will depend on the strength of gravitational field at the orbital radius.

Consider the following situations. Which identical satellite will be travelling faster in each case? Why?

a) Satellite A orbits the Earth at twice the orbital radius of Satellite B.

b/c it is closer ∴ Fg is greater so it must move faster to stay in orbit

b) Satellite A orbits the Sun at the same orbital radius that Satellite B orbits the Earth.

same reason. Sun has more mass so Sat A must move faster to stay in orbit

The orbital period of the satellite depends only on the mass of the planet and the orbital radius of the satellite. It stands to reason that at a certain orbital distance the orbital period will match the rotational period of the planet. Such a satellite is said to be in geosynchronous (or geostationary) orbit.

Example: (a) Find the orbital radius of a satellite that is geostationary above Earth's equator.

$T_{Earth} = 24\,h \times \frac{60\,min}{h} \times \frac{60\,s}{min} = 86400\,s$

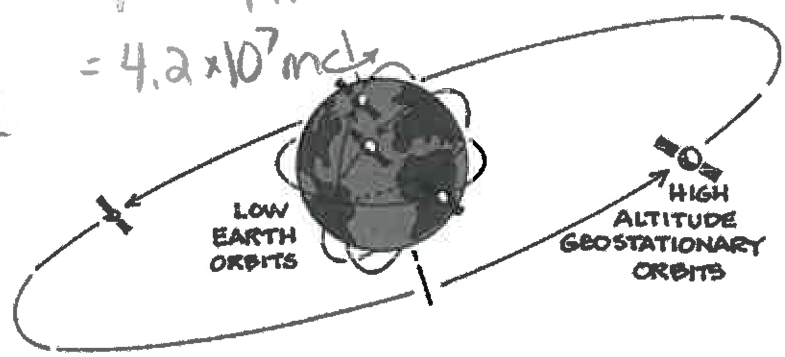
$a_c = g$

$\frac{4\pi^2 r}{T^2} = \frac{Gm_1}{r^2} \rightarrow 4\pi^2 r = \frac{T^2 G m_1}{r^2}$

$r^3 = \frac{T^2 G m_1}{4\pi^2}$

$r = \sqrt[3]{\frac{(86400)^2(6.67\times10^{-11})(5.98\times10^{24}\,kg)}{4\pi^2}}$

$= 4.2\times10^7\,m$



(b) What is the speed of this satellite?

$v = \frac{2\pi r}{T} = \frac{2\pi(4.22\times10^7)}{86400}$

$= 3072\,m/s$

Text Q's p.302 #1,2,3

p.303 #2,4