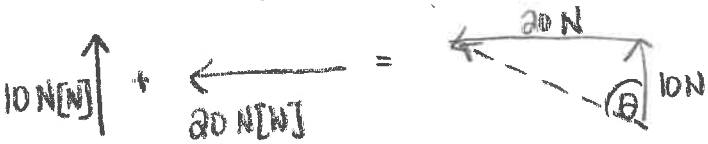
# Adding Vectors
(continued)

3. Pythagorean Method

- used for perpendicular vectors (i.e. vectors form a right triangle)
- use the Pythagorean Theorem ($R = \sqrt{X^2 + Y^2}$) to find the magnitude of the resultant
- solve for direction using trig

Example: A block of wood sits on a desk. Student A pushes it with 10.0 N north and Student B pushes with 20.0 N west. What is the total force exerted on the block?




$R = \sqrt{x^2 + y^2}$

$= \sqrt{20^2 + 10^2}$

$= 22.36 = 22.4 N$

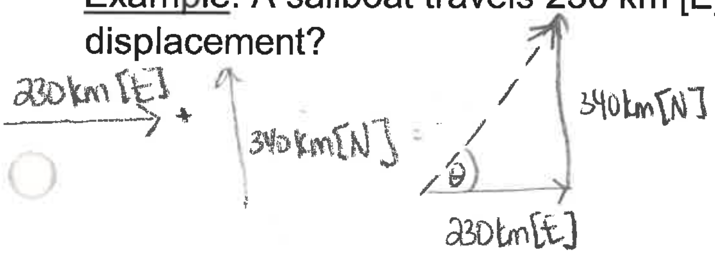
$\tan\theta = \frac{O}{A}$

$\theta = \tan^{-1}\left(\frac{20}{10}\right)$

$= 63°$

$\boxed{\vec{F}_R = 22.4 N [N 63° W]}$

Example: A sailboat travels 230 km [E] and then 340 km [N]. What is the resultant displacement?




$R = \sqrt{x^2 + y^2}$

$= \sqrt{230^2 + 340^2}$

$= 410.5 = 410 km$

$\theta = \tan^{-1}\left(\frac{340}{230}\right)$

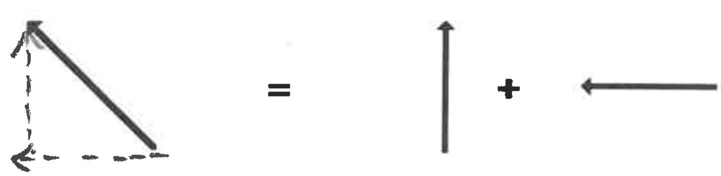
$= 56°$

watch θ; convert

$\boxed{\vec{d}_R = 410 km [N 34° E]}$

**Background: Vector Components**

Any vector directed in two dimensions can be thought of as having an influence in two different directions. That is, it can be thought of as having two parts. Each part of a two-dimensional vector is known as a component.

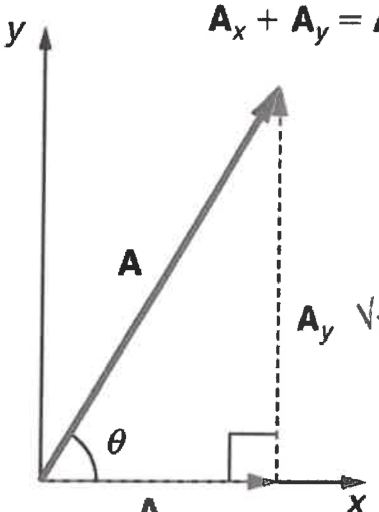



**Northwest vectors have a northward and westward part.**


**An upward and rightward vector has an upward and rightward part.**

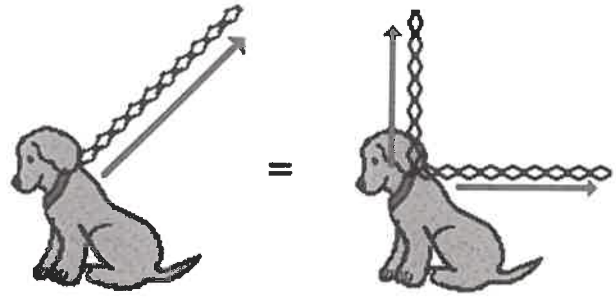
For example: Vector A

$A_x + A_y = A$

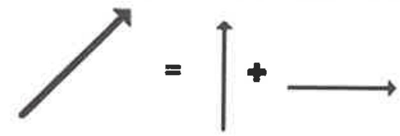



vertical(y) component

horizontal(x) component



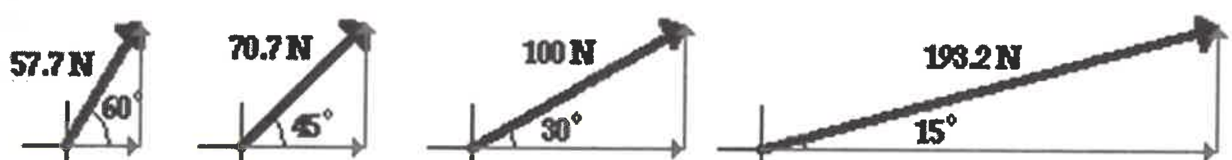
A pull upon Fido's chain in an upward and a rightward direction exerts two separate influences upon Fido - an upward and a rightward influence.



The force exerted at the angle is equal to the vector sum of the two individual forces.

The two components of a vector are independent of each other. A change in one component does NOT affect the other component.

While the change in one of the components will alter the magnitude of the resultant, it does not alter the magnitude of the other component.




*a change in x component affects magnitude of R but not y component*

**Each of the four vectors above has the same vertical component of force - 50 N. The four vectors have different horizontal components of force. Altering the horizontal component will affect the horizontal moton of the object to which this force is applied.**

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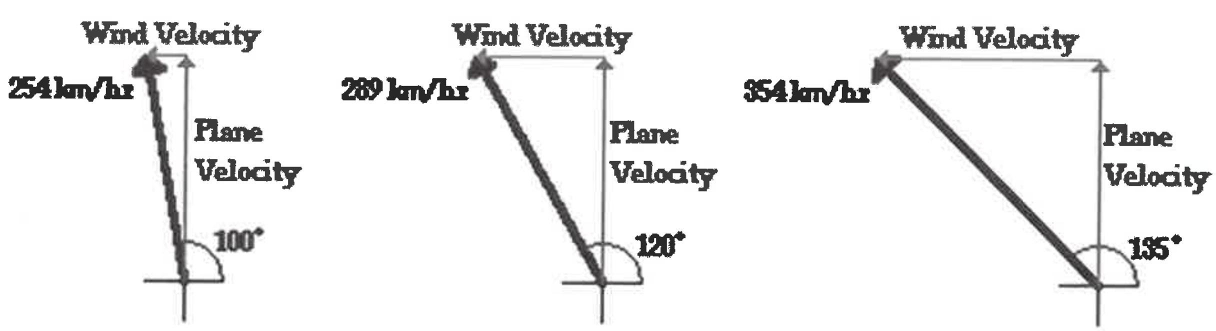



$V_{wind}$ = 44 km/hr
$V_{plane}$ = 250 km/hr

$V_{wind}$ = 144 km/hr
$V_{plane}$ = 250 km/hr

$V_{wind}$ = 250 km/hr
$V_{plane}$ = 250 km/hr

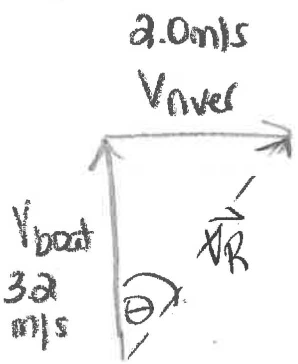
**The resulting motion of a plane in the presence of a wind is dependent upon the velocity of the crosswind. An alteration of the wind velocity affects the resulting motion but does NOT affect the velocity at which the plane flies northward. Perpendicular components of motion are independent of each other.**

Ex: A student in a canoe is trying to cross a 45 m wide river that flows due East at 2.0 m/s. The student can paddle at 3.2 m/s

a. If he points due North and paddles how long will it take him to cross the river?

$V_y = \frac{d_y}{t}$ $\quad t = \frac{d_y}{V_y} = \frac{45m}{3.2m/s} = \boxed{14s}$

b. What is his total velocity relative to his starting point in part a?




$V_R = \sqrt{V_{river}^2 + V_{boat}^2}$

$= \sqrt{(2^2) + (3.2^2)}$

$= 3.77 m/s = 3.8 m/s$

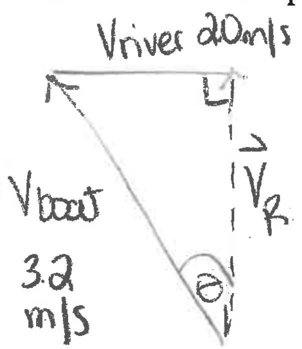
$\theta = \tan^{-1}\left(\frac{O}{A}\right)$

$= \tan^{-1}\left(\frac{2}{3.2}\right)$

$= 32°$

$\boxed{\vec{V_R} = 3.8 m/s [N 32° E]}$

c. If he needs to end up directly North across the river from his starting point, what heading should he take?




$\sin\theta = \frac{O}{H}$

$\theta = \sin^{-1}\left(\frac{2}{3.2}\right)$

$= 39°$ or $\boxed{N 39° W}$

d. How long will it take him to cross the river at this heading?

$V_{boat}^2 = V_{river}^2 + V_R^2$

$V_R^2 = V_{boat}^2 - V_{river}^2$

$V_R = \sqrt{(3.2)^2 - (2)^2}$

$V_R = 2.5 m/s$ $\quad V_y = \frac{d_y}{t}$

$t = \frac{d_y}{V_y} = \frac{45m}{2.50m/s} = \boxed{18s}$