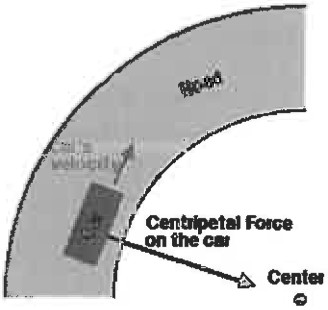
Turning a Car, Banked Curves and 2 D Problems



Let's review... As a car makes an unbanked (flat) turn, the force of friction between the tires and the road provides the centripetal force required for circular motion.

Static friction

Example: A 1.0 x 10^3 kg car rounds a curve on a flat road with a radius of 50.0 meters at a constant speed of 50.0 km/hr. Will the car follow the curve or...?

a. On dry pavement with the coefficient of static friction of 0.60

Given: m = 1000 kg
r = 50 m
convert → v = 50 km/hr = 13.89 m/s
$\mu = 0.60$
Need: r

$F_C = \frac{mv^2}{r}$

$F_S = \frac{mv^2}{r}$

$\mu \cdot F_N = \frac{mv^2}{r}$

$r = \frac{mv^2}{\mu \cdot mg}$ × masses cancel

$= \frac{v^2}{\mu \cdot g} = \frac{(13.89\ m/s)^2}{0.60 \cdot 9.8\ m/s^2} = 32.81\ m$

= so yes it will follow the curve! 33 m < 50 m radius

b. In icy conditions when the coefficient of static friction becomes 0.25

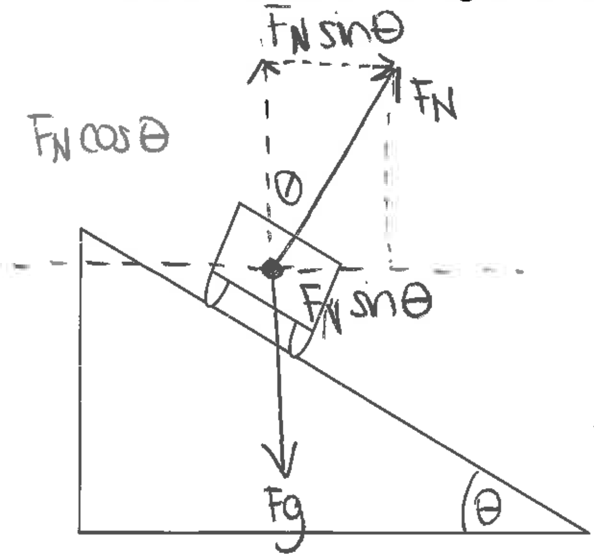
$r = \frac{v^2}{\mu \cdot g} = \frac{(13.9\ m/s)^2}{0.25 \cdot 9.8\ m/s^2} = 78.75\ m > 50\ m$ radius so it will not follow the curve!

What happens if the force of friction is insufficient? The car will skid (see picture) aka go straight (Newton's First law)

Is it possible to still make a turn if no friction exists? Yes, we can bank the curve

Banking the curve can keep the car from skidding because...
the normal force now has a horizontal component which can act as the centripetal force

Consider a car traveling at a constant speed around a frictionless banked corner.



In the vertical, there is no acceleration so...

$F_N\cos\theta = mg \qquad F_N = \dfrac{mg}{\cos\theta}$

In the horizontal...

$F_C = F_N\sin\theta$

$= \left(\dfrac{mg}{\cos\theta}\right)\sin\theta \quad \left(\dfrac{\frac{O}{H}}{\frac{A}{H}} = \dfrac{O\cdot H}{H\cdot A} = \tan\theta\right)$

Since $F_{net} = F_C$ $= mg\tan\theta$

$mg\tan\theta = \dfrac{mv^2}{r}$

Solving for v gives:

$v = \sqrt{rg\tan\theta}$

meaning that a car moving at v will successfully round the curve!

Example: A curve has a radius of 50 meters and a banking angle of 15º. What is the ideal, or critical, speed (the speed for which no friction is required between the car's tires and the surface) for a car on this curve?

Given: $r = 50m$
$\theta = 15°$

Need: v

$v = \sqrt{rg\tan\theta}$

$= \sqrt{50m \cdot 9.8\frac{m}{s^2}\tan 15}$

$\pm = 11.46 = \boxed{11\,m/s}$

alternate

$\tan\theta = \dfrac{F_c}{F_g}$

$\tan\theta = \dfrac{\frac{mv^2}{r}}{mg}$

$v^2 = rg\tan\theta$

$v = \sqrt{rg\tan\theta}$

Example: Calculate the angle at which a frictionless curve must be banked if a car is to round it safely at a speed of 22 m/s if its radius is 475 m.

Given: $v = 22\,m/s$
$r = 475m$

Need: θ

$\tan\theta = \dfrac{v^2}{rg}$

$\theta = \tan^{-1}\left(\dfrac{v^2}{rg}\right)$

$= \tan^{-1}\left(\dfrac{(22m/s)^2}{475\cdot 9.8}\right)$

$= 5.94° = \boxed{6°}$