4.0 The Solubility Product Constant

Since saturated solutions are equilibrium systems, we can apply the equilibrium expression as follows:

$$Ag_2SO_{4\,(s)} \rightleftharpoons 2Ag^+_{(aq)} + SO_4^{2-}{}_{(aq)}$$

$$K_{sp} = \frac{[Ag^+]^2[SO_4^{2-}]}{[Ag_2SO_4]}$$

where our equilibrium constant is now called K_{sp}, where "sp" stands for "**s**olubility **p**roduct".

Don't forget that in equilibrium expressions solids and liquids are NOT included.

Therefore: $K_{sp} = [Ag^+]^2[SO_4^{2-}]$

Example: Write the expression for the solubility product constant, K_{sp}, for $Ca_3(PO_4)_2$.

Solution: Write the dissolving reaction (remember that polyatomic ions remain together as a unit and do not break apart into separate elements). Write the K_{sp} expression.

$$Ca_3(PO_4)_{2\,(s)} \rightleftharpoons 3\,Ca^{2+}_{(aq)} + 2\,PO_4^{3-}{}_{(aq)}$$

$$K_{sp} = [Ca^{2+}]^3[PO_4^{3-}]^2$$

4.1 The Meaning of K_{sp}

The K_{sp} is a measure of the solubility of an ionic salt.

The larger the value of the K_{sp}, the greater is the solubility of the salt.

- FeS has a K_{sp} of 4×10^{-19} which means that in a saturated solution there would be few Fe^{2+} or S^{2-} ions.
- $PbCl_2$ has a K_{sp} of 1.8×10^{-4} which means a saturated solution of $PbCl_2$ would have many Pb^{2+} and Cl^- ions.

Don't forget that you can only calculate a K_{sp} if the solution is saturated. Only saturated salt solutions are in equilibrium.

4.2 Calculations Involving K_{sp}

(a) … when concentration of a saturated solution is known

Example: The concentration of lead ions in a saturated solution of PbI_2 at 25°C is 1.30×10^{-3} M. What is its K_{sp}?

Solution: $PbI_{2\,(s)} \rightleftharpoons Pb^{2+}_{(aq)} + 2\,I^-_{(aq)}$

$K_{sp} = [Pb^{2+}][I^-]^2$

Now use the equation ratio to find the concentration of the ions.
$[PbI_2] = 1.30 \times 10^{-3}$M
$[Pb^{2+}] = 1.30 \times 10^{-3}$M
$[I^-] = 2 \times 1.30 \times 10^{-3} = 2.60 \times 10^{-3}$ M

Substitute values into the K_{sp} expression and solve for the unknown:

$K_{sp} = [Pb^{2+}][I^-]^2$

$= (1.30 \times 10^{-3})(2.60 \times 10^{-3})^2$

$= (1.30 \times 10^{-3})(6.76 \times 10^{-6})$

$K_{sp} = 8.79 \times 10^{-9}$M

(b) … when the amount of solute required to saturate a solution is given

Example: Calculate the Ksp for $CaCl_2$ if 2.00×10^2g of $CaCl_2$ is required to saturate 100.0 mL of solution.

Solution: $CaCl_{2\,(s)} \rightleftarrows Ca^{2+}_{(aq)} + 2Cl^-_{(aq)}$

$Ksp = [Ca^{2+}][Cl^-]^2$

First calculate molarity.

$$\text{Molarity} = \frac{200\ g \times \frac{1\ mole}{111.1\ g}}{0.100L} = 18.001\ M$$

Now use the equation ratio to find the concentration of the ions.
$[CaCl_2] = 18.0\ M$
$[Ca^{2+}] = 18.0\ M$
$[Cl^-] = 36.0M$

Substitute values into the K_{sp} expression and solve for the unknown:

$Ksp = [18.0][36.0]^2$
$Ksp = 2.33 \times 10^4\ M$

(c) … when K_{sp} is known.

<u>Example</u>: K_{sp} for $MgCO_3$ at 25°C is 2.0×10^{-8}. What are the ion concentrations in a saturated solution at this temperature?

Solution: $MgCO_{3(s)} \rightleftharpoons Mg^{2+}_{(aq)} + CO_3^{2-}{}_{(aq)}$

$$K_{sp} = [Mg^{2+}][CO_3^{2-}]$$

We will let our unknown ion concentrations equal x. The balanced equation tells us that both Mg^{2+} and $CO3^{2-}$ will have the same concentration!

Substitute values into the equation and solve for the unknown:

$$K_{sp} = [Mg^{2+}][CO_3^{2-}]$$
$$2.0 \times 10^{-8} = (x)\ (x) = x^2$$
$$x^2 = 2.0 \times 10^{-8}$$
$$x = \sqrt{2.0 \times 10^{-8}}$$
$$= 1.4 \times 10^{-4} M$$
$$x = [Mg^{2+}] = 1.4 \times 10^{-4} M$$
$$x = [CO_3^{2-}] = 1.4 \times 10^{-4} M$$