Electric Fields

Just like a gravitational field, electric fields exist and depend on size (of the charge) and the distance (between charges).

An electric field is defined as the force per unit charge and can be calculated from:

$$\vec{E} = \frac{F_E}{q}$$

← just lik grav fields $g = \frac{F_g}{m}$

Where E = electric field, measured in N/C
F_E = electric force, measured in N
q = test charge, measured in C

Example: What is the electric field strength at a point where a 2.00 uC charge experiences an electric force of 5.30×10^{-4} N?

$$\vec{E} = \frac{F_E}{q} = \frac{5.30\times10^{-4}\,N}{2.00\times10^{-6}\,C} = 265\,N/C$$

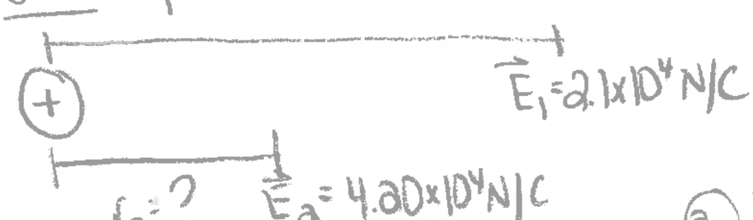
Adding Coulomb's Law, we now find the electric field strength as:

$$\vec{E} = \frac{F_E}{q} \quad \therefore F_E = \vec{E}q \qquad F_E = \frac{Kq_1q_2}{r^2} \qquad \therefore \vec{E}q = \frac{Kq_1q_2}{r^2}$$

$$\vec{E} = \frac{Kq_2}{r^2}$$

Example: At a distance of 7.50×10^{-1} m from a small charged object the electric field strength is 2.10×10^4 N/C. At what distance from this same object would the electric field strength be 4.20×10^4 N/C?

Given: $r_1 = 0.750\,m$



solve for q from E_1, then use E_2 + q to find r_2

① $\vec{E}_1 = \frac{Kq}{r_1^2}$ $\quad q = \frac{r^2\vec{E}}{K} = \frac{(0.750\,m)^2(2.1\times10^4\,N/C)}{8.99\times10^9\,N\cdot m^2/C^2} = 1.313\times10^{-6}\,C$

② $E_2 = \frac{Kq}{r_2^2}$

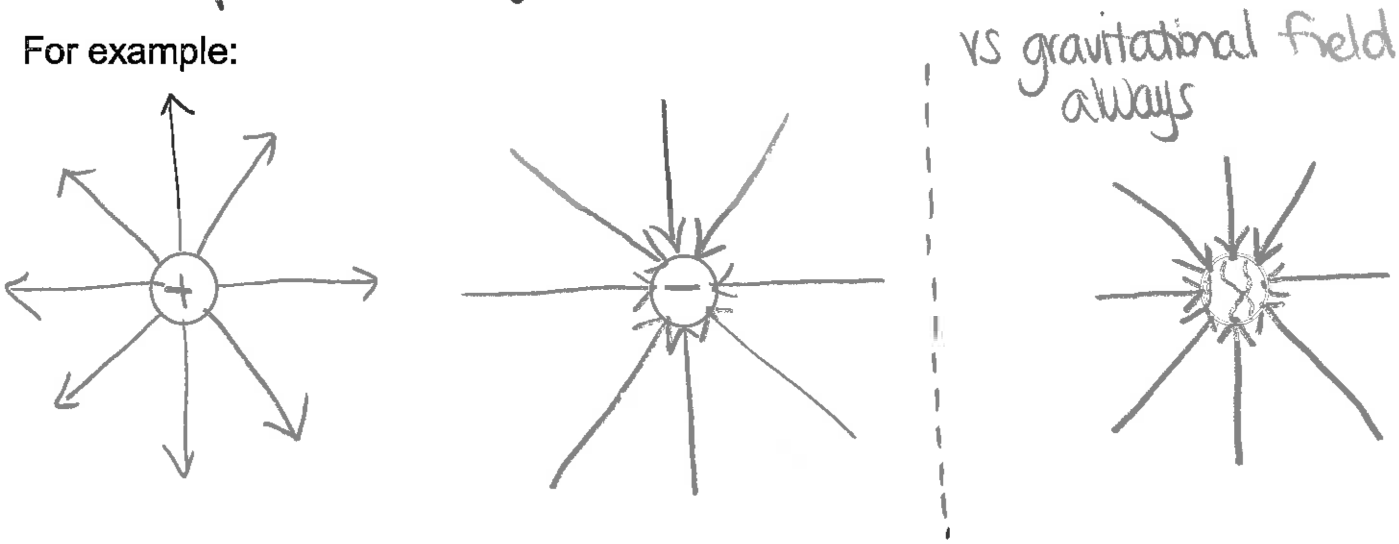
$r_2^2 = \frac{Kq}{E_2}$

$r = \sqrt{(8.99\times10^9)(1.313\times10^{-6})/4.20\times10^4} = 0.530\,m$

Just like with gravitational fields, electric fields are vector quantities whose field lines are represented by arrows.

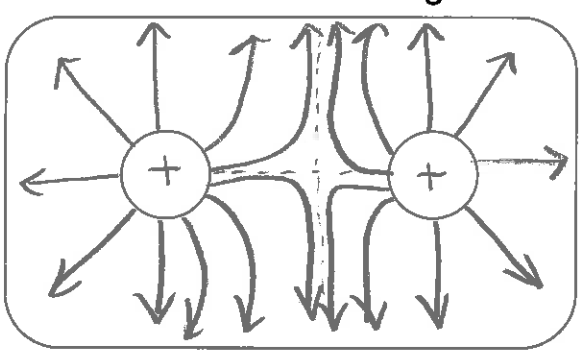
One important difference: the direction of the electric field is the direction a positive charge would move in that field

For example:

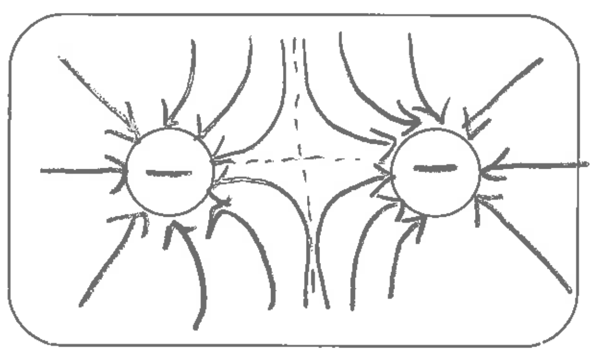


These field lines work for individual charges, but what happens when another charge is introduced?

Two Positive Charges

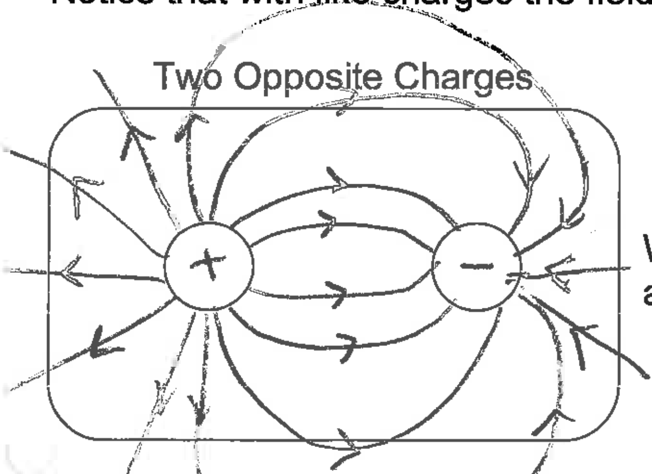


Two Negative Charges



Notice that with like charges the fields interact to work in opposition to each other.

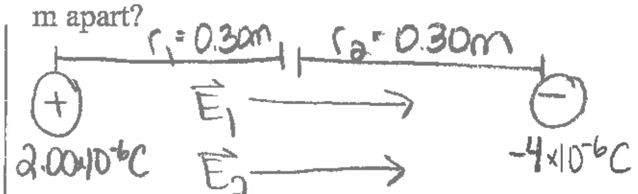
Two Opposite Charges



With opposite charges, the lines of force work together and reinforce each other.

Since electric fields are force fields and vector quantities, when multiple fields overlap we can simply add them up as vectors (paying attention to attraction/repulsion though!).

Example:
What is the strength of an electric field midway between a 2.00 uC charge and a -4.00 uC that are 0.60 m apart?

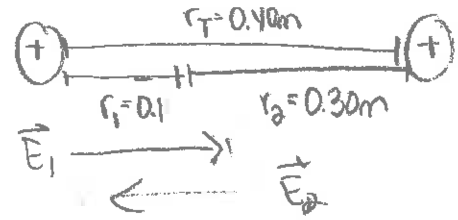


$E_1 = \frac{kq_1}{r_1^2} = \frac{(8.99\times10^9)(2\times10^{-6})}{(0.3)^2} = 2.0\times10^5$ N/C

$E_2 = \frac{kq_1}{r_2^2} = 4.0\times10^5$ N/C *don't write the -ve sign for the charge*

$E_T = \vec{E}_1 + \vec{E}_2 = 6.00\times10^5$ N/C towards the -ve

Example:
Two 5.25 uC charges are 0.40 m apart. What is the strength of the electric field between them at a point 0.10 m away from the first charge and 0.30 m away from the second?



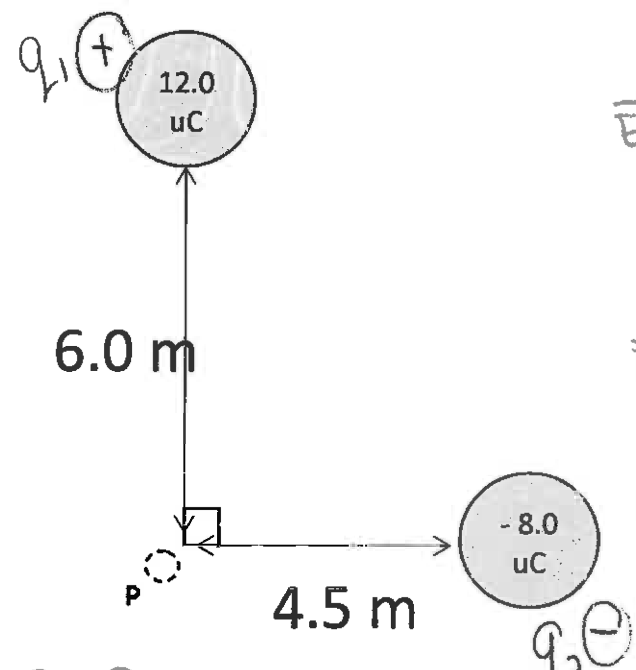
$E_T = E_1 - E_2$

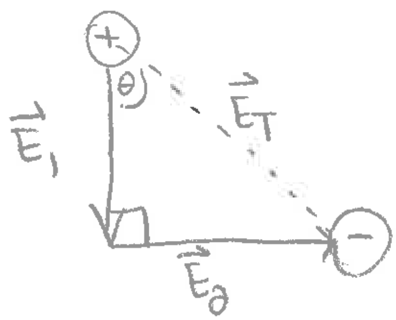
$= \frac{kq_1}{r_1^2} - \frac{kq_2}{r_2^2}$

$= \frac{(8.99\times10^9)(5.25\times10^{-6})}{(0.1m)^2} - \frac{(8.99\times10^9)(5.25\times10^{-6})}{(0.30)^2}$

$= 4190000 = 4.2\times10^6$ N/C

Example:
Find the magnitude and direction of the electric field at the point P due to the charges as shown. Suppose that a proton was placed at point P. What would its initial acceleration be?





$\vec{E}_1 = \frac{kq_1}{r_1^2} = 3000$ N/C

$\vec{E}_2 = \frac{kq_2}{r_2^2} = 3556$ N/C

$E_T = \sqrt{E_1^2 + E_2^2}$

$= 4650$ N/C

$\tan\theta = \frac{E_2}{E_1}$

$\theta = \tan^{-1}\left(\frac{\vec{E}_2}{\vec{E}_1}\right)$

$= 50°$

$\vec{E}_T = \frac{F_E}{q}$

$F_E = E_T \cdot q$

$= 4650 N/C \cdot 1.60\times10^{-19}C$

$= 7.44\times10^{-16}N$

$a = \frac{F}{m}$ ($m = 1.67\times10^{-27}$ kg)

$= 4.5\times10^{11}$ m/s^2