

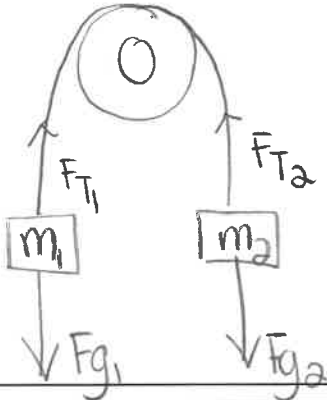
## Dynamics Notes

### Two Objects and Tension

There are a number of common force problems that involve 2 objects and tension, beyond the basic two body problems. We will focus on 3 of these.

#### 1) Two Masses Suspended By a Pulley

Diagram: Include all forces at work on the two masses.



Both masses have a  $F_g$  that pull downwards, but since they are connected by a pulley those forces work in opposite to each other.

The masses will accelerate so that the... larger one wins!

Since they are attached by a rope the acceleration of the masses must be

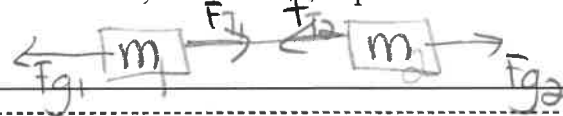
EQUAL.

Note that there is a force of *TENSION* (T) that exists along the rope. Tension acts the same as all other forces, with two important peculiarities:

- 1) It is an internal force, acting... in both directions at any point along the rope
  - 2) It cancels out... of the  $F_{net} (\Sigma F)$  equation
- (when using systems approach)

#### The Strategies:

- When solving these problems it is easiest for us to choose the direction ... of the heavier mass winning as positive
- Remember that the acceleration on the two masses... must be equal
- It can also be easier to conceptualize this problem if we "unfold" the masses and lay them out in a line, while keeping all of our forces as they are... I know that sounds weird so, here's an example.



Ex  
Two masses are suspended from a lightweight rope over a frictionless pulley as shown. What will their acceleration be once released?

$$\Sigma F_x = ma$$

$$F_{g1} - F_{g2} = ma$$

$$a = \frac{F_{g1} - F_{g2}}{m_T} = \frac{8 \text{ kg}(9.8 \text{ m/s}^2) - 5 \text{ kg}(9.8 \text{ m/s}^2)}{13 \text{ kg}}$$

$$= 2.26 = \boxed{2.3 \text{ m/s}^2}$$

**NOTE:** When calculating the acceleration we use the total mass because the  $F_{net}$  is accelerating the entire system (both masses)!

Alright that wasn't too hard, but can you find the tension in the rope?

If we use the same force diagrams and equations as before we hit a snag. The two tension forces cancel !!!

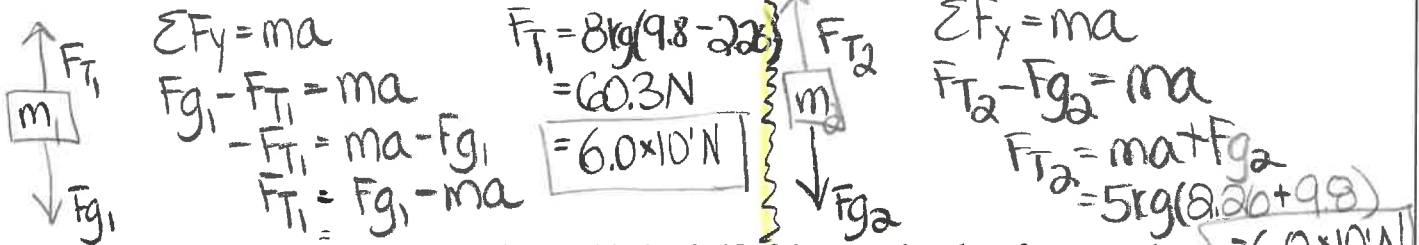
This is because tension is an internal force.

In order to solve for tension we have to consider... one mass only!

**Strategy:** To solve for tension chop your diagram in half and only consider one of the masses. Either one is fine because...

both tensions are **EQUAL!**

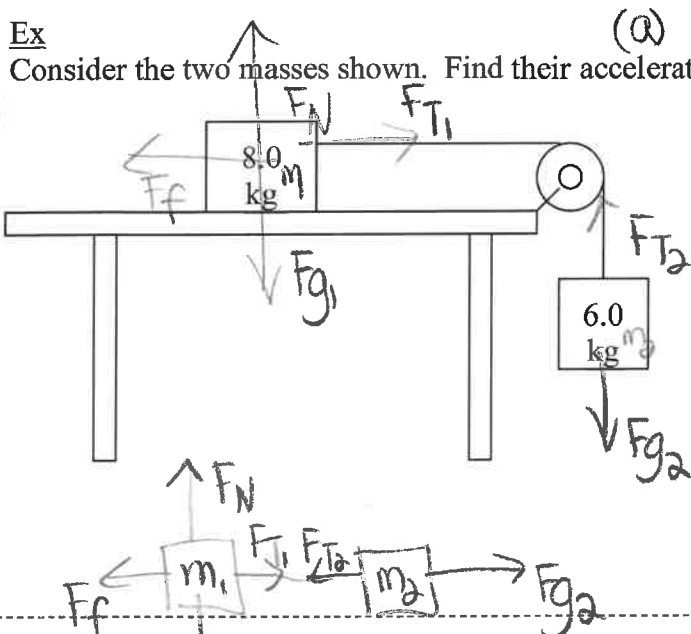
**Ex** - Find the tension in the rope in the preceding example.



**Note:** When finding the tension we are only considering half of the equation therefore we only use individual mass!

## 2) The Hanging Mass

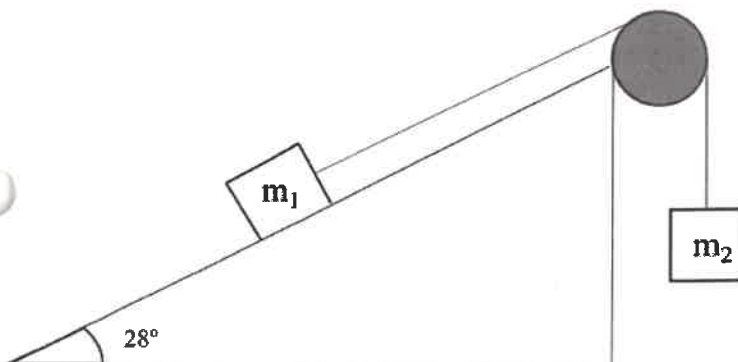
**Ex**  
Consider the two masses shown. Find their acceleration and the tension in the rope if  $\mu_{\text{table}} = 0.21$ .



(a)  $\Sigma F_x = ma$   
 $F_{g2} - F_f = ma$   
 $a = \frac{F_{g2} - F_f}{m_T}$   
 $= \frac{m_2 g - \mu m_1 g}{m_T}$   
 $= \frac{6 \text{ kg}(9.8 \text{ m/s}^2) - 0.21(8.0 \text{ kg})(9.8 \text{ m/s}^2)}{14 \text{ kg}}$   
 $= 3.02 = 3.0 \text{ m/s}^2$

(b)  $\Sigma F_y = ma$   
 $F_{g2} - F_{T2} = ma$   
 $-F_{T2} = ma - F_{g2}$   
 $F_{T2} = F_{g2} - ma$   
 $= 40.68$   
 $= 41 \text{ N}$

## 3) Two Masses On an Incline



**Problem:** What direction will they accelerate?

In earlier pulley problems it was obvious, the bigger mass always wins. When an inclined plane is involved this is not always the case because for the mass on an incline only...

the parallel component of  $F_g$  counts!

Ex

Two forces are attached by a rope over a frictionless pulley as shown.

(Assume the incline is frictionless) Determine:

- a. The acceleration of the masses.
- b. The tension in the rope.

(a)  $\Sigma F_x = ma$

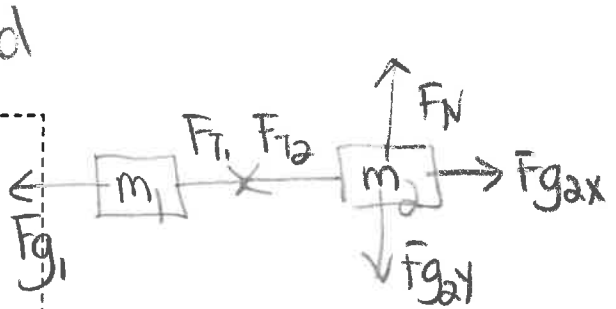
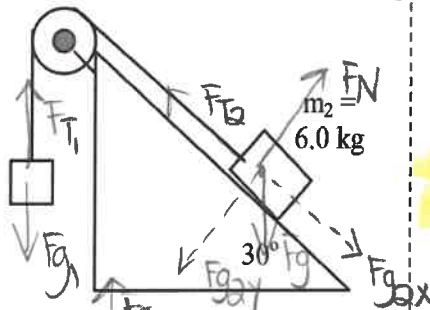
$F_{g1} - F_{gax} = ma$

$a = \frac{F_{g1} - F_{gax}}{m_T}$

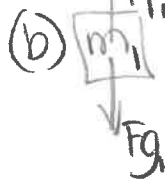
$= \frac{39.2 - 29.4}{10 \text{ kg}}$

$a = 0.98 \text{ m/s}^2$

$m_1 = 4.0 \text{ kg}$



So determine the forces on each one separately and see which one is the winner.



(b)  $\Sigma F_y = ma$   
 $F_{g1} = m_1 g = 4 \text{ kg} \cdot 9.8 \frac{\text{m}}{\text{s}^2} = 39.2 \text{ N}$   
 $F_{g1} - F_{T1} = ma$   
 $-F_{T1} = ma - F_{g1} = 39.2 \text{ N}$   
 $F_{T1} = F_{g1} - ma$   
 $= 39.2 \text{ N} - 4(0.98)$   
**Winner!**

Force 2 =  $F_{gax}$

$F_{gax} = m_2 g \sin \theta$   
 $= 6 \text{ kg} / (9.8 \text{ m/s}^2) \sin 30^\circ$   
 $= 29.4 \text{ N}$

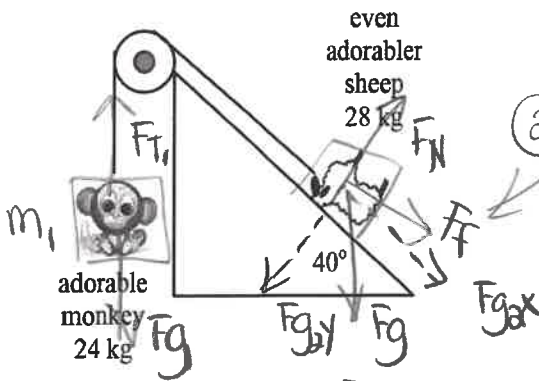
$F_T = 35 \text{ N}$

Ex

In the name of physics, a monkey is attached to a sleeping sheep on a ramp. Don't ask why.

As we all know, the coefficient of friction for a sleeping sheep on a ramp is precisely 0.15. Determine:

- a. The acceleration of the system.
- b. The tension in the rope.

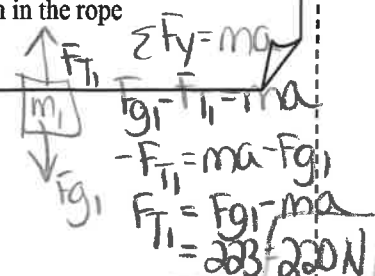


① Find Winners/Losers  
 $F_{g1} = m_1 g = 235.2 \text{ N}$   
 $F_{gax} = m_2 g \sin 40 = 176.4 \text{ N}$   
 $F_{g1}$  wins! meaning  
 $F_f$  acts down the ramp

$F_f = \mu F_N$   
 $= \mu m g \cos 40$   
 $= 0.15 (28)(9.8) \cos 40$   
 $= 31.5 \text{ N}$

**Strategies:**

1. Find the forces acting on the two bodies separately to determine a winner
2. Determine the friction on the sheep. Friction can work either up or down the ramp, because it always opposes motion, so we don't know which direction it is acting until we know the winner.
3. Based on the winner find the acceleration using  $m_{total}$ . Choose either body and examine it separately to determine the tension in the rope



$\Sigma F_x = ma$   
 $F_{g1} - F_f - F_{gax} = ma$

$a = \frac{F_{g1} - F_f - F_{gax}}{m_T} = \frac{235.2 \text{ N} - 31.5 \text{ N} - 176.4 \text{ N}}{24 + 28 \text{ kg}} = 0.525 = 0.53 \frac{\text{m}}{\text{s}^2}$

