Conservation of Energy

When only conservative forces (like gravity) work on an object...

total mechanical energy is conserved ($E_p \rightarrow E_k$, $E_k \rightarrow E_p$)

$$E_T = E_g + E_K$$

For situations where only conservative forces are involved:

$$E_{T_i} = E_{T_f}$$

$$E_{g_i} + E_{k_i} = E_{g_f} + E_{k_f}$$

$$mg\Delta y_i + \frac{1}{2}mv_i^2 = mg\Delta y_2 + \frac{1}{2}mv_f^2$$

Example: While jumping over The Great Wall of China an 82 kg skateboarder is needs to leave the ramp traveling at 78 km/h.

a. How much potential energy does he need to start with?

Given: $m = 82\,kg$

$v_f = 78\,\frac{km}{h}(\div 3.6) = 21.67\,\frac{m}{s}$

$E_{k_i} = 0$

$E_{g_f} = 0$ (arbitrary)

Need: E_{g_i}

$$E_{T_i} = E_{T_f}$$

$$E_{g_i} + E_{k_i} = E_{g_f} + E_{k_f}$$

$$E_{g_i} = E_{k_f}$$

$$= \frac{1}{2}mv_f^2$$

$$= \frac{1}{2}(82\,kg)\left(\frac{21.67\,m}{s}\right)^2$$

$$= 19247 = \boxed{19000\,J}$$

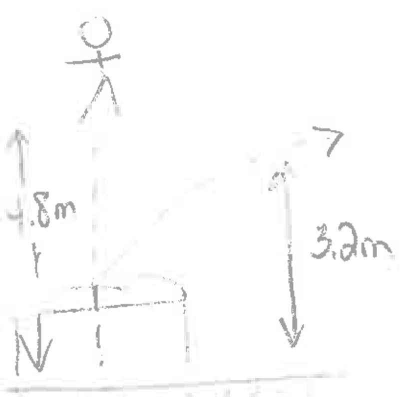
b. What minimum height of ramp should he use?

$$E_g = mg\Delta y$$

$$\Delta y = \frac{E_g}{mg} = \frac{19247\,J}{(82)(9.8)} = \boxed{24\,m}$$

Example: A trampoline dunk artist is bounces to a maximum vertical height of 4.8 m before launching himself towards the hoop. At the top of his arc he is 3.2 m above the ground. How fast is he traveling at this point?

assume starts at rest



$$E_{T_i} = E_{T_f}$$

$$E_{g_i} + \cancel{E_{k_i}} = E_{g_f} + E_{k_f}$$

$$mg\Delta y_i + \frac{1}{2}mv_i^2 = mg\Delta y_f + \frac{1}{2}mv_f^2 \quad (\times 2; \div \text{by } m)$$

$$2g\Delta y_i + \cancel{v_i^2} = 2g\Delta y_f + v_f^2$$

$$v_f^2 = 2g(\Delta y_i - \Delta y_f)$$

$$v_f = \sqrt{2(9.8)(4.8-3.2)} = \boxed{5.6\,m/s}$$

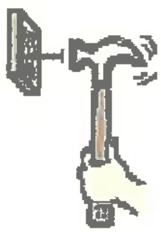
When nonconservative forces (like friction, air resistance) are at work, then the total mechanical energy is changed.

If the work is positive work, then the object will ___gain___ energy.

If the work is negative work, then the object will ___lose___ energy.

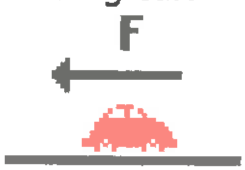
Example:

a. Rusty Nales pounds a nail into a block of wood. The hammer head is moving horizontally when it applies force to the nail.



← $force_a$ = Δd are in same direction ∴ +ve work
← Δd applied force ↑ E_k of the nail

b. The frictional force between highway and tires pushes backwards on the tires of a skidding car.



← $force_f$ + Δd are opp. directions ∴ -ve work
→ Δd friction causes car to ↓ E_k
* energy is not "lost" goes to heat (changes out of mechanical forms)

c. A weightlifter applies a force to lift a barbell above his head at constant speed.

↑ ↑
$force_a$ Δd ∴ +ve work
applied force causes barbell to ↑ Δy
∴ ↑ E_g